



JUNE – 2025

PART – B

338. Let $f: \mathbb{R} \setminus \mathbb{Q} \rightarrow \mathbb{R} \setminus \mathbb{Q}$ be the function defined as $f(x) = \frac{3x+2}{4x+3}$.

Let $x_1 \in \mathbb{R} \setminus \mathbb{Q}$. For $n \geq 1$, define $x_{n+1} = f(x_n)$. Suppose that the sequence $(x_n)_{n \geq 1}$ converges to a real number ℓ . Which of the following statements is true?

(1) If ℓ is positive, the $\ell = \frac{\sqrt{3}}{2}$.

(2) If ℓ is positive, then $\ell = \frac{1}{\sqrt{2}}$.

(3) If ℓ is negative, then $\ell = -\frac{\sqrt{3}}{2}$.

(4) If ℓ is negative, then $\ell = -\frac{1}{2}$.

339. Let p, q be non-negative integers. Consider the following statements:

- (A) There is an integer $k \geq 1$ such that $p + k = q$.
(B) There is an integer $k \geq 1$ such that $q + k = p$

Which of the following statements is true?

- (1) There exist non-negative integers p, q such that both (A) and (B) are true.
(2) Both (A) and (B) are false if and only if $p = q$
(3) For all non-negative integers p and q , (A) or (B) is true
(4) There exists $p \neq q$ such that both (A) and (B) are false

340. Let $f(x) = x \log_e \left(1 + \frac{1}{x}\right)$ for $x \in (0, \infty)$. Which of the following statements is true?

(1) f is unbounded

(2) f is increasing

(3) $\lim_{x \rightarrow \infty} f(x) = 2$

(4) f is decreasing

341. For each $n \geq 1$, let $f_n: [0, 1] \rightarrow \mathbb{R}$ be defined as

$$f_n(x) = \begin{cases} nx & \text{if } x \in \left[0, \frac{1}{n}\right], \\ 2 - nx & \text{if } x \in \left(\frac{1}{n}, \frac{2}{n}\right], \\ 0 & \text{if } x \in \left(\frac{2}{n}, 1\right]. \end{cases}$$

Which of the following statements is true?

- (1) $(f_n)_{n \geq 1}$ converges uniformly on $[0, 1]$ to a continuous function f .
(2) $(f_n)_{n \geq 1}$ converges pointwise on $[0, 1]$ to a discontinuous function f .
(3) $(f_n)_{n \geq 1}$ converges pointwise on $[0, 1]$ to a continuous function f .
(4) $(f_n)_{n \geq 1}$ does not converge pointwise on $[0, 1]$.

342. Let A, B be non-empty subsets of $\mathbb{N}$ with cardinality $|A| \geq 2$. Let $S_1 = \{f: A \rightarrow B \mid f \text{ is one-to-one}\}$ and $S_2 = \{g: B \rightarrow A \mid g \text{ is onto}\}$. Which of the following statements is true?

- (1) If $A \subsetneq B$ and B is finite, then there is a one-to-one map from S_2 to S_1 .
(2) If $B = \mathbb{N}$, then there exists a one-to-one map from S_2 to B .



(3) If $B = \mathbb{N}$ and A is finite, then there exists a one-to-one map from B to S_1 .

(4) If A is finite, then S_2 is finite for any B .

343. Let

$$A = \left\{ \frac{p}{q} \in (0,1) : p \in \mathbb{N}, q = 2^n \text{ for some } n \in \mathbb{N} \cup \{0\}, \gcd(p, q) = 1 \right\}$$

$$B = \left\{ \frac{p}{q} \in (0,1) : p \in \mathbb{N}, q = 2^n 5^m \text{ for some } n, m \in \mathbb{N} \cup \{0\}, \gcd(p, q) = 1 \right\}$$

$$C = \left\{ \frac{p}{q} \in (0,1) : \frac{p}{q} \text{ has terminating decimal expansion} \right\}$$

be subsets of $(0, 1)$. Which of the following statements is true?

(1) $A \subsetneq C$ and $B \subsetneq C$

(2) $A \subsetneq C \subsetneq B$

(3) $A \subsetneq B \subsetneq C$

(4) $A \subsetneq B = C$

PART-C

344. Let f and g be real-valued Riemann integrable functions on $[a, b]$ such that $g([a, b]) \subseteq [a, b]$. Which of the following statements are necessarily true?

1. The composition $f \circ g$ is Riemann integrable.

2. If $g(x) \neq 0$ for each $x \in [a, b]$, then $\frac{f}{g}$ is Riemann integrable.

3. The positive square root $\sqrt{f^2 + g^2}$ is Riemann integrable.

4. The composition $f \circ g$ is Riemann integrable, if both f and g are continuous

345. Let $C[0, 1]$ be the $\mathbb{R}$ -vector space of real valued continuous functions equipped with the norm

$$\|f\| = \sup_{x \in [0,1]} |f(x)|.$$

Let $T : C[0,1] \rightarrow C[0,1]$ be defined as

$$T(f)(x) = \int_0^x f(t) dt, \text{ for } x \in [0,1]$$

Let $T^n = T \circ T \circ \dots \circ T$ (n times). Which of the following statements are true?

1. There exists $\alpha \in (0,1)$ such that for all $f, g \in C[0,1]$, $\|T(f) - T(g)\| \leq \alpha \|f - g\|$.

2. There exists $\alpha \in (0,1)$ such that for all $f, g \in C[0,1]$, $\|T^2(f) - T^2(g)\| \leq \alpha \|f - g\|$.

3. The set $\{f \in C[0,1] : T(f) = f\}$ is a singleton set.

4. $\|T^n\| \rightarrow \infty$ as $n \rightarrow \infty$.

346. Let f be a bounded, twice continuously differentiable real-valued function on $(0, \infty)$ such that $f''(x) \geq 0$ for all $x \in (0, \infty)$. Which of the following statements are true?

1. $f'(x) \leq 0$ for all $x > 0$

2. $\lim_{x \rightarrow \infty} f'(x) = 0$

3. $\lim_{x \rightarrow \infty} x f'(x)$ need not exist.

4. $\lim_{x \rightarrow \infty} x f'(x) = 0$



347. Consider the sequence $(s_n)_{n \geq 1}$ and $(t_n)_{n \geq 1}$ defined by

$$s_n = \sum_{k=0}^n \frac{1}{(k!)^2} \text{ and } t_n = \sum_{k=0}^n \binom{n}{k} \frac{(-1)^k}{(n)^k}.$$

Which of the following statements are true?

1. $\limsup_{n \rightarrow \infty} t_n \leq \limsup_{n \rightarrow \infty} s_n$
2. $\limsup_{n \rightarrow \infty} t_n \leq e$
3. $\liminf_{n \rightarrow \infty} s_n \geq e^2$
4. $\liminf_{n \rightarrow \infty} t_n \geq e$

348. Let μ denote the Lebesgue measure on $\mathbb{R}$. Suppose that f is a non-negative Lebesgue measurable function on $\mathbb{R}$. Let $0 = a_0 < a_1 < a_2 < \dots$ be an unbounded sequence such that $a_{n+1} \leq ca_n$ for some real number c and for all $n \geq 1$. Let $A_k = \{x \in \mathbb{R} \mid a_k \leq f(x) < a_{k+1}\}$ for each $k \geq 0$. Which of the following statements are true?

1. If f is Lebesgue integrable on $\mathbb{R}$, then $\sum_{k \geq 0} a_k \mu(A_k)$ is finite.
2. If $\sum_{k \geq 0} a_k \mu(A_k)$ is finite, then f is Lebesgue integrable on $\mathbb{R}$.
3. If $\sum_{k \geq 0} a_k \mu(A_k)$ is finite and $f(x) \geq \alpha$ for all $x \in \mathbb{R}$, then f is Lebesgue integrable on $\mathbb{R}$.
4. If $\sum_{k \geq 0} a_k \mu(A_k)$ is finite and f is bounded, then f is Lebesgue integrable on $\mathbb{R}$.

349. Let $p: \mathbb{R} \rightarrow \mathbb{R}$ be a non-constant polynomial. Which of the following statements are true?

1. The pre-image of a compact set under p is a compact set.
2. The pre-image of a connected set under p is a connected set.
3. Every point $x \in \mathbb{R}$ has an open neighbourhood U_x such that the restriction $p|_{U_x}$ is a homeomorphism onto an open set in $\mathbb{R}$.
4. The image of a bounded set under p is a bounded set.

350. For each $n \geq 1$, let $f_n: \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by $f_n(x) = \frac{e^{-n^2 x^2}}{n}$.

Which of the following statements are true?

1. $(f_n)_{n \geq 1}$ converges uniformly to 0 on $\mathbb{R}$, and $(f'_n)_{n \geq 1}$ converges uniformly to 0 on the interval $(-M, M)$ for some positive real number M .
2. $(f_n)_{n \geq 1}$ converges uniformly to 0 on $\mathbb{R}$, and $(f'_n)_{n \geq 1}$ converges pointwise to 0 on $\mathbb{R}$.
3. $(f_n)_{n \geq 1}$ converges uniformly to 0 on $\mathbb{R}$, and $(f'_n)_{n \geq 1}$ does not converge pointwise to 0 on $\mathbb{R}$.
4. $(f_n)_{n \geq 1}$ converges pointwise to 0 on $\mathbb{R}$ but not uniformly on $\mathbb{R}$.

351. What is the value of the limit

$$\lim_{n \rightarrow \infty} \frac{1}{n} [(n+1)(n+2) \cdots (n+n)]^{1/n} ?$$

1. $\frac{2}{e}$
2. $\frac{4}{e}$
3. $\log_e 2 - 1$
4. $2 \log_e 2 - 1$



ANSWERS

- | | | | | | |
|----------------|----------------|----------------|--------------|----------------|----------------|
| 1. (1) | 2. (4) | 3. (3) | 4. (2) | 5. | 6. (3) |
| 7. (4) | 8. (1,3) | 9. (1,3,4) | 10. (3,4) | 11. | 12. (4) |
| 13. (3,4) | 14. (3,4) | 15. (1,2) | 16. (3,4) | 17. (1,2,3) | 18. (2,4) |
| 19. (3) | 20. (3) | 21. (4) | 22. (2) | 23. (2) | 24. (2) |
| 25. (1) | 26. (3) | 27. (1,2) | 28. (1,2,4) | 29. (1,2,3) | 30. (1,2) |
| 31. (1,2) | 32. (2,3,4) | 33. (1) | 34. (1,2,4) | 35. (3) | 36. (3) |
| 37. (1) | 38. (1) | 39. (4) | 40. (3) | 41. (2,4) | 42. (3,4) |
| 43. (2,3) | 44. (1,4) | 45. (1,3,4) | 46. (1,4) | 47. (2,4) | 48. (3) |
| 49. (3) | 50. (2) | 51. (1) | 52. (4) | 53. (2) | 54. (1,2,3,4) |
| 55. (1,3) | 56. (3,4) | 57. (1,2) | 58. (1,3) | 59. (3,4) | 60. (1,2) |
| 61. (1,2,3) | 62. (2,4) | 63. (4) | 64. (3,4) | 65. (3,4) | 66. (1) |
| 67. (1) | 68. (1) | 69. (2) | 70. (3) | 71. (1) | 72. (1,4) |
| 73. (3,4) | 74. (3,4) | 75. (2,3) | 76. (3,4) | 77. (2,4) | 78. (1,4) |
| 79. (2,4) | 80. (2) | 81. (1) | 82. (2) | 83. (1) | 84. (2) |
| 85. (3) | 86. (1,2) | 87. (1) | 88. (1,4) | 89. (1,2,3) | 90. (1,2,4) |
| 91. (1,2,4) | 92. (1,2) | 93. (1,3,4) | 94. (1,3) | 95. (1) | 96. (1,4) |
| 97. (1) | 98. (4) | 99. (3) | 100. (3) | 101. (3) | 102. (1) |
| 103. (3) | 104. (2) | 105. (2,4) | 106. (2,3) | 107. (3) | 108. (1,2,3) |
| 109. (3,4) | 110. (3,4) | 111. (2) | 112. (3) | 113. (3) | 114. (2) |
| 115. (3) | 116. (2) | 117. (2) | 118. (3) | 119. (2) | 120. (2) |
| 121. (3) | 122. (2) | 123. (1) | 124. (1,3,4) | 125. (3,4) | 126. (3,4) |
| 127. (4) | 128. (2,3,4) | 129. (2) | 130. (1,2,3) | 131. (1,4) | 132. (1,2,3,4) |
| 133. (1,3) | 134. (2) | 135. (4) | 136. (2) | 137. (1) | 138. (2) |
| 139. (3) | 140. (3) | 141. (3) | 142. (2) | 143. (1,4) | 144. (1,2, 4) |
| 145. (1,2) | 146. (1,2) | 147. (1,4) | 148. (1,2) | 149. (4) | 150. (1,3,4) |
| 151. (2) | 152. (1,3) | 153. (3) | 154. (2) | 155. (4) | 156. (3) |
| 157. (4) | 158. (2) | 159. (2,3) | 160. (1,4) | 161. (3,4) | 162. (1,2) |
| 163. (1,2,3,4) | 164. (2,3) | 165. (1,2,4) | 166. (1,2,3) | 167. (2,4) | 168. (2) |
| 169. (2) | 170. (4) | 171. (2) | 172. (2) | 173. (1) | 174. (2,3) |
| 175. (2,4) | 176. (1,2,3,4) | 177. (1,2) | 178. (2,3) | 179. (2,3) | 180. (1,3) |
| 181. (1,2,3) | 182. (3,4) | 183. (1,2,3,4) | 184. (2) | 185. (3) | 186. (3) |
| 187. (2) | 188. (3) | 189. (2) | 190. (1,2,4) | 191. (2,3,4) | 192. (3) |
| 193. (1,4) | 194. (1,3,4) | 195. (4) | 196. (1) | 197. (4) | 198. (3,4) |
| 199. (1,2,4) | 200. (3,4) | 201. (1,2,3,4) | 202. (3) | 203. (3) | 204. (4) |
| 205. (4) | 206. (4) | 207. (3) | 208. (2,3) | 209. (1,4) | 210. (1,4) |
| 211. (1,4) | 212. (1,3) | 213. (2) | 214. (2,3,4) | 215. (2,3,4) | 216. (2) |
| 217. (1,2,3) | 218. (1,3) | 219. (1) | 220. (2) | 221. (3) | 222. (3) |
| 223. (4) | 224. (2) | 225. (4) | 226. (1) | 227. (2,3) | 228. (2,4) |
| 229. (3,4) | 230. (1,4) | 231. (3,4) | 232. (4) | 233. (1,2) | 234. (1,3,4) |
| 235. (3) | 236. | 237. (3) | 238. (2) | 239. (3) | 240. (4) |
| 241. (1) | 242. (2) | 243. (1) | 244. (1) | 245. (2,4) | 246. (3,4) |
| 247. (3) | 248. (1,2) | 249. (1,3) | 250. (1,2) | 251. (1,3) | 252. (1,2,3,4) |
| 253. (4) | 254. (3) | 255. (3) | 256. (2) | 257. (4) | 258. (2) |
| 259. (3) | 260. (3) | 261. (4) | 262. (1,4) | 263. (1) | 264. (2,3) |
| 265. (3) | 266. (1,3,4) | 267. (1,4) | 268. (1,2,4) | 269. (3) | 270. (1,3,4) |
| 271. (2) | 272. (2) | 273. (3) | 274. (3) | 275. (1) | 276. (2) |
| 277. (3) | 278. (3,4) | 279. (2) | 280. (2) | 281. (1,3) | 282. (1,2,3,4) |
| 283. (1,2,3,4) | 284. (2,3,4) | 285. (1,4) | 286. (1,2) | 287. (2,3) | 288. (2) |
| 289. (3) | 290. (3) | 291. (1) | 292. (3) | 293. (2) | 294. (2) |
| 295. (1,2) | 296. (1,2) | 297. (1,3) | 298. (1,3,4) | 299. (1,2,4) | 300. (1,2,3,4) |
| 301. (1,3,4) | 302. (2,3,4) | 303. (1,4) | 304. (4) | 305. (1,2,3) | 306. (3) |
| 307. (2) | 308. (2) | 309. (3) | 310. (4) | 311. (3) | 312. (4) |
| 313. (1,2) | 314. (3) | 315. (1,2,4) | 316. (1,3) | 317. (1,2,3,4) | 318. (1,4) |
| 319. (2,3) | 320. (3,4) | 321. (3) | 322. (1,2,4) | 323. (3) | 324. (1) |
| 325. (4) | 326. (3) | 327. (4) | 328. (1,2,4) | 329. (1,2,4) | 330. (1,2,3,4) |